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# Application of Efficient Global Optimization in Ship Mechanics

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Abstract: Efficient Global Optimization (EGO), one Bayesian analysis optimization algorithm, makes use of Kriging model to construct statistical approximation model and uses infill sampling criteria (ISC) to find the next sampling point updating the model. This method is discussed in detail and applied in ship mechanics with two traditional optimization examples. One is a submarine conceptual multidisciplinary design, which considers hydrodynamics, propulsion, weight and volume, performance, and cost. It is a mixed- variable optimization problem defined by 8 real design variables,3 integer design variables and 12 constraints. The other is stiffened panel optimization under buckling. Compared with traditional methods, EGO not only finds the global optimal point, but also completes the optimization more efficiently. The results demonstrate that EGO is very suitable for optimization in ship mechanics. Key words: Efficient Global Optimization (EGO); ship mechanics; Kriging model;

infill sampling criteria (ISC)

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# 1 Introduction

Modern engineering design problems are complex and involve multiple disciplines, as the same in the field of ship mechanics. In order to reduce the complexity and improve the design efficiency, the multidisciplinary design optimization (MDO)<sup>[1]</sup> is brought into engineering design field to solve such large coupled systems. However, during the application of MDO in ship mechanics, the expense of analysis models is often a prime concern, because some disciplinary analyses need lots of time to operate, such as CFD for hydrodynamics and FEA for structural analysis. If we integrate these disciplinary analyses into the overall design optimization directly, the computation is tremendous. Optimization with surrogate modeling, which can reduce computational time and improve the efficiency, seems to be a good substitute, but it just finds the approximate optimal value, or even the local optimal value as the imprecision of the surrogate modeling. Constructing a precise metamodel with large amount of sample points, is a tremendous work ahead of optimization.

Efficient Global Optimization (EGO), developed by Jones, Schonlau and Welch<sup>12</sup>, belongs to Bayesian analysis optimization algorithms. It constructs Kriging model with a small initial

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data sample within the design space. Based on this model, a so-called infill sampling criterion is evaluated to select a set of additional points, and then the approximation is updated with these points to improve accuracy. This process continues until the improvement expected from sampling additional points has become sufficiently small. Comparing with other surrogate model, EGO not only converges to global optimal value, but also works efficiently.

For the purpose of exploring how EGO can be applied in the Ship Mechanics, this paper presents two examples. One is a submarine conceptual multidisciplinary design, which is a mixed-variable optimization problem. The other is stiffened panel optimization under buckling. Finally, the EGO results are compared with results using common method to demonstrate the advantages of EGO.

## 2 Efficient Global Optimization

Efficient Global Optimization (EGO), one of global optimization, fits into a general class of optimization algorithms which we will refer to as Bayesian analysis algorithms<sup>[3]</sup>. These global searching algorithms use statistical models from an initial data sample to determine where to evaluate the next functions via an auxiliary optimization problem. EGO uses Kriging model as the statistical approximation model, and uses a generalized expected improvement function to prefer the points that either low objective function value or high uncertainty. 2.1 Kriging model

Kriging is named after D.G. Krige, a South African geologist, who first used the statisticsbased technique in analyzing mining data in the 1950s and 1960s<sup>[4]</sup>. It assumes response value is the weighted sum of sample data while the weighting coefficients are obtained through the best liner unbiased prediction (BLUP). Since the end of 1980s, Kriging model was developed as a new direction applied to deterministic computer experiments<sup>[5]</sup>. This Kriging model concentrates and specializes on only a small portion of the available knowledge, extremely flexible due to the wide range of correlation functions chosen for constructing the approximation model. This paper adopts the second kriging model to build metamodel.

The present Kriging model combines a global model plus localized departures:

$$\mathbf{y}(\mathbf{x}) = \mathbf{f}(\mathbf{x}) + \mathbf{Z}(\mathbf{x}) \tag{1}$$

where y(x) is the unknown function of interest, f(x) is the known global model (usually polynomial) to approximate the design space, and Z(x) is the realization of a stochastic process with mean zero and nonzero covariance to represent a local deviation from the global model. The correlation between  $Z(x^{i})$  and  $Z(x^{j})$  is strongly related to the distance between the two corresponding points,  $x^{i}$  and  $x^{j}$ . In EGO, a special weighted distance function between the points  $x^{i}$  and  $x^{j}$  is expressed as

$$d(\mathbf{x}^{i},\mathbf{x}^{j}) = \sum_{h=1}^{k} \theta_{h} |\mathbf{x}_{h}^{i} - \mathbf{x}_{h}^{j}|^{P_{h}} \qquad (\theta_{h} \quad \mathbf{0}, \mathbf{p}_{h} \quad [\mathbf{1},\mathbf{2}])$$
(2)

where  $\theta_h$  are the unknown correlation parameters used to fit the model,  $p_h$  represent the smoothness of the function in coordinate direction h. With this distance function, the correlation between the points  $x^i$  and  $x^j$  is defined as

$$\operatorname{Cov}[Z(\mathbf{x}^{i}), Z(\mathbf{x}^{j})] = \exp[-d(\mathbf{x}^{i}, \mathbf{x}^{j})]$$
(3)

The Kriging model predicts the response value as

$$\hat{\mathbf{y}} = \hat{\boldsymbol{\beta}} + \mathbf{r}^{\mathsf{T}}(\mathbf{x}) \, \mathbf{R}^{-1} \left( \mathbf{y} - \mathbf{1} \hat{\boldsymbol{\beta}} \right) \tag{4}$$

where  $\hat{\beta}$  is the estimated value of f using Eq.(5):

$$\hat{\beta} = \frac{1}{1} \frac{R^{-1} y}{R^{-1} 1}$$
(5)

R denotes the n xn matrix whose (i, j) entry is  $Cov[Z(x^{i}), Z(x^{j})]$ , r is the correlation vector whose  $i_{th}$  element is

$$\mathbf{r}_{i}(\mathbf{x}) = \mathbf{Cov}[\mathbf{Z}(\mathbf{x}), \mathbf{Z}(\mathbf{x}^{i})]$$
 (6)

and 1 denotes an n-vector of ones.

The unknown parameter of Eq.(4) is  $\theta$ , which can obtained by maximizing the following likelihood function

$$Ln(\theta) = -\frac{n}{2} ln(2\pi) - \frac{n}{2} ln(\hat{\sigma}^{2}) - \frac{1}{2} ln(|R|) - \frac{1}{2\hat{\sigma}^{2}} (y - 1\hat{\beta})^{T} R^{-1} (y - 1\hat{\beta})$$
(7)

where  $\hat{\sigma}^{\,\,^{2}}$  can be defined as

$$\hat{\sigma}^{2} = \frac{\left(\mathbf{y} - \mathbf{1}\hat{\boldsymbol{\beta}}\right)^{\mathsf{T}} \mathsf{R}^{-1} \left(\mathbf{y} - \mathbf{1}\hat{\boldsymbol{\beta}}\right)}{\mathsf{n}}$$
(8)

when  $\theta$  is obtained, the Kriging model can be obtained using Eqs.(4)~(6). 2.2 Infill Sampling Criteria (ISC)

The infill sampling criteria determines which design points to sample next (the so-called infill samples). It is a completely different method than algorithms that rely on a search path because the sampling criterion could place the next iteration anywhere at all in the design space. Furthermore, it tends to choose the design points most likely to improve the accuracy of the model and/or have a better function value than the current best point. The ISC used by EGO is known as the expected improvement function, which is defined as

where  $f_{min}$  is the minimum feasible sampled value of function after n evaluations,  $\hat{y}$  and  $\hat{\sigma}$  are estimated response and variance using Kriging model.,  $\Phi(\cdot)$  and  $\phi(\cdot)$  denote the cumulative distribution function and probability density function of the standard normal distribution, respectively.

The first term in Eq.(9) tends to be large where  $\hat{y}$  is likely smaller than  $f_{min}$ , and the sec-

ond term tends to be large where there is high uncertainty about whether or not  $\hat{y}$  will be better than  $f_{min}$ . Therefore, the expected improvement will tend to be large which is likely improved and to be of high uncertainty.

In order to determine the next sample points to be evaluated, the ISC problem will be maximized as follows:

max ISC=EI

s.t. g(x) = 0

2.3 The procedure of EGO

The basic procedure of EGO is as follows:

(1) Use a space-filling design of experiments to create a set of initial sample points and evaluate the response at sample points;

(2) Based upon the sample points and their evaluations, construct approximation model with Kriging model;

(3) Perform one complete optimization with infill sampling criteria to search for the point where to sample;

(4) Calculate response of the true function at the new sample point and update the Kriging model with this sample point;

(5) Check if the expected improvement function has become sufficiently small. If sufficiently small, terminate the process. Otherwise, return to (2).

During the step (2),one issue should be paid attention to. If the approximation model does not fit well, it is always to improve the model by appropriate transformations of the response. Generally, cross validation, a statistical technique, is often used to assess the predictive capability of the approximation model. When the model is found to be bad fit, the response function will be transforming with the log transformation,  $\ln(y)$  or  $-\ln(-y)$ , or the inverse transformation, -1/y.

## 3 Numerical examples

In order to demonstrate EGO well suitable for ship mechanics, this paper adopts two examples. One is a submarine conceptual multidisciplinary design, which is a mixed-variable optimization problem; the other is stiffened panel optimization under buckling, and the numerical analysis is carried out using ANSYS.

3.1 Implementation to a submarine conceptual design optimization with EGO

The design of a submarine is a complex, multidisciplinary process which is characterized by thousand of design variables, multi-objectives and nonlinear constraints. A complete design requires analyses of hydrodynamics, propulsion, weight and volume, performance, cost and the others. It is important that each of these aspects should be addressed at the conceptual design

(10)

phase. In this application, the main disciplines of hydrodynamics, propulsion, weight and volume, performance, and cost are included. The hydrodynamics analysis is responsible for predicting the physics of motion and action of the submarine in water, the state output from this analysis is an estimate of total resistance, envelope volume and some hydrodynamics characteristics of the submarine; the propulsion analysis mainly considers the torque of the electric motor which is transmitted into the thrust on the hull, and calculates the propulsion system weight, volume and power characteristics based on the database of propulsion plant types and battery types; Weight prediction is based on empirical models developed from a database of submarine of this designed and fabricated series during the past decade, and determines total submarine weight and center of gravity location; performance analysis mainly estimates maximum endurance speed (sprint speed), sprint range and endurance range; cost analysis estimates the basic construction cost of the submarine.

The optimization problem, defined by 8 real design variables, 3 integer design variables and 12 constraints, is summarized in Tab.1.

Design variables	Constraints	Objective function
Length forebody (Lf)	Sprint speed > Required sprint speed	
Length midbody (Lm)	Sprint range > Required sprint range	
Lenght aftbody (La)	Endurance range > Required endurance range	
Diameter of vessel (D)	Prepare power > Required power	Basic
Operation depth	Prepare space > Required space	Construction
Battery energy	Maximum free flood volume required ratio> 0	Cost
Fuel weight	Minimum free flood volume required ratio> 0	
Endurance speed	Minimum lead required ratio > 0	
* Power plant type	Maximum lead required ratio > 0	
* Battery type	Minimum GM required ratio > 0	
* Number of generator	Minimum GB required ratio > 0	
	Primary electric power required ratio > 0	

Tab.1 Summary of the submarine conceptual design problem

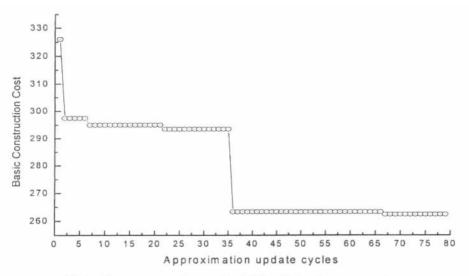
(\*: the discrete variable)

Tab.2 shows the optimization result with EGO, meanwhile, compared with results for traditional MDO method, Multidisciplinary Design Feasible (MDF) and Collaborative Optimization (CO), which is given in [6]. The results for three different optimization methods are presented for the comparison purpose. Hereinto, EGO has found the global optimal point, and the optimization result with EGO is obviously better than two other results. The optimization convergence history of the objective with EGO is shown in Fig.1, which demonstrates that optimization problem is close to convergence after 36 cycles and converges at the 79th cycle. Cao Anxi et al<sup>[1]</sup> spent lots of computation in finding an appropriate initial point, or the optimization would fail to converge. Besides, both MDF and CO did not find the global optimal point. Compared with these two methods, EGO not only finds the global optimal point, bus also completes the optimization with less evaluations of the objective.

Parameters	Traditi	ional optimization (MDF)	Collabora	tive Optimization		EGO	
Geometry size	[8.01,16	[8.01,16.53,16.68,6.76]m		[7.92,17,16.58,6.79]m		[7.64,14.10,16.45,7.25]m	
[Lf, Lm, La, D]							
Operation depth	76.2	m	76.23	m	76.2	m	
Battery energy	8 540	kwhr	8 460	kwhr	9 366	kwhr	
Fuel weight	10.861	t	11.287	t	7.064	t	
Endurance speed	10.6	knots	10.45	knots	10	knots	
* Power plant type	1	type	1	type	1	type	
* Battery type	1	type	1	type	1	type	
* Number of generator	1	number	1	number	1	number	
Basic construction cost	263.562	million dollars	269.144 5	million dollars	262.394 6	million dollars	

Tab.2 Result of the submarine optimization

(\*: the discrete variable)





#### 3.2 Stiffened panel optimization under buckling

Stiffened panels are basic elements of all types of ship structures, and absorb lateral loads and distribute those loads to the ship 's primary structures, so the overall improvement in ship structures is mainly dependent on the improved design of these panels. Light weight and high strength are the major objectives for the design of stiffened panels, but they are sometimes contradicted, because the reduction of weight may lead to the reduction of structure strength. Therefore, it is meaningful to optimize the stiffened panels for the purpose of improvement in ship structures. For traditional design, empirical formulas can be used to evaluate the response of stiffened panel, but they may be less useful for new type stiffened panels, so the designer has to rely more on numerical simulation codes used for optimization design. However, since the numerical simulation codes are used in the optimization design, the derivatives of objective and constraints are hard to obtain, and the responses of simulation are usually noisy or nonsmooth. Therefore, it is hard to optimize this kind of problem with the optimization method based on derivative, such as Sequential Quadratic Programming (NLPQL) and Method of Feasible Directions (CONMIN), which are sensitive to initial point and sometimes converge to the partial optimal point. Optimization with approximation may be a good way to improve the efficiency, but it needs a lot of sample points to construct an accurate surrogate model before optimization, or the optimization result is doubtful. Therefore, EGO is chosen to optimize this problem.

Since buckling of stiffened panels has been a topic of interest for many years, this example considers stiffened panel optimization under buckling. The design objective is to minimize the mass of stiffened panel with the satisfactory of buckling load requirement. Tab.3 summarizes the optimization problem. Besides, the width and length of stiffened panel both are 3 600mm, and the thickness of all plates in 15mm. The web height, the flange width and the location of the side stiffener with ranges of [50 150], [100 200] and [400 1 400] (mm) are taken as the independent variables, the axial pressure is applied on the two transverse edges which are simply supported and have rotational restraint about the z-axis, as can be shown in Fig.2.

1 4010			
Design variables	Constant	Constraints	Objective function
Web height (W_H)	Young 's modulus of E= 205.8GPa	Eigenvalue buckling load	
Flange width (F_W)	Poisson 's ratio of $\nu$ =0.3	<	Mass of stiffened panel
Location of the side stiffener (L_S)		Required buckling load	

Tab.3 Summary of stiffened panel optimization design problem

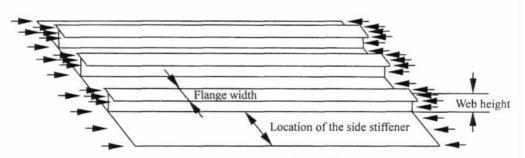


Fig.2 Stiffened panel model

For simplicity,only the eigenvalue buckling load is considered in this example to compare the efficiency and the accuracy between EGO and Multi-Island Genetic Algorithm (MIGA)<sup>[7]</sup>, the numerical simulation is carried out using ANSYS.

MIGA is a distributed Genetic Algorithm (GA) and more efficient than traditional GA. The main feature of this method is that each population of individuals in one generation is divided into several sub-populations called 'Island'. All traditional genetic operations are performed independently on each sub-population, and then some individuals are selected from each island and migrated to different islands periodically. This method overcomes the premature convergence of traditional GA and enables the calculation to converge global optimal solutions. The MIGA used in this optimization case is performed with iSIGHT<sup>[7]</sup>.

Before the optimization, we make some modification for EGO. Since the constraint of this problem is obtained through ANSYS simulation and not cheaply to be computed, we create a penalty function with this constraint. Therefore, this case turns to be an unconstrained opti-

mization problem, and the penalty function can be written as follows.

 $\mathbf{F}(\mathbf{x}) = \mathbf{f}(\mathbf{x}) + \boldsymbol{\omega} \cdot \left(\max(\mathbf{0}, \mathbf{g}(\mathbf{x}))\right)^2$ (11)

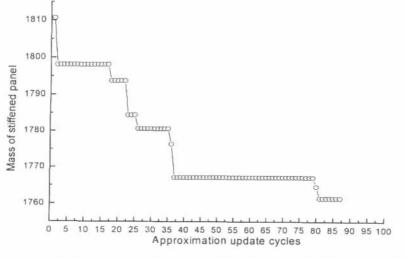
where the f(x) is the mass of stiffened panel, $\omega$  is the penalty factor,and g(x) is the constraint as following.

Tab.4 summarizes the results for the EGO and MIGA.Both methods find the identical design, within a slight tolerance. We can see that EGO finds the optimal point just with 87 simulations, while Multi-Island Genetic Algorithm takes 1 000 simulations to find the optimal point. Besides, the EGO 's optimization procedure takes about 11.6 minutes, while the MIGA takes about 84.6 minutes.

Parameters	EGO	MIGA
Web height (W_H)	0.085 1m	0.084 7m
Flange width (F_W)	0.100 0m	0.100 Om
Location of the side stiffener (L_S)	1.016 6m	1.063 4m
Mass of stiffened panel	1 761.448 2kg	1 760.990 0kg
Number of simulations	87	1 000
Total time (minute)	11.6	84.6

Tab.4 Result of the submarine optimization

Fig.3 shows the optimization convergence history of the objective with EGO, we can see that the optimization result with Kriging model, which is constructed by 33 sample points using Latin Hybercube, is not the global optimal value, because the initial Kriging model is not accurate enough. After 80 cycles of ANSYS simulation, the EGO found the optimization result and converged at 87th cycle.Fig.4 demonstrates the optimization convergence history of the objective with MIGA.It shows that obtaining a point nearby the global optimal point needs 8 generations, which means needing ANSYS to simulate 400 times. Furthermore, the optimal value is found after 20 generations, which means needing ANSYS to simulate 1 000 times.





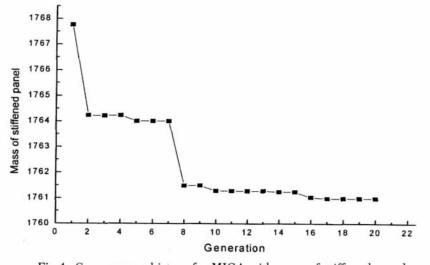


Fig.4 Convergence history for MIGA with mass of stiffened panel

In this case, EGO not only shows perfect global search ability, but also performs more efficiently than MIGA. It can be deducted that EGO is more suitable for optimization expensive (Costly) and time- consuming global optimization. In ship structural mechanics, FEM softwares are used in order to provide high-fidelity predictions for the structural responses, while the computation always is expensive (Costly) and time- consuming. Therefore, comparing with traditional method, EGO is more suitable for the optimization in ship structural mechanics.

#### 4 Summary and conclusions

Efficient Global Optimization is the most appealing method of global optimization. This method is discussed in detail in this paper, and applied in ship mechanics with two examples. One is a submarine conceptual multidisciplinary design, which is a mixed-variable optimization problem defined by 8 real design variables, 3 integer design variables and 12 constraints, the other is stiffened panel optimization under buckling. Both are typical optimization problems in ship mechanics. Compared with traditional method, EGO not only finds the global optimal point, but also completes the optimization more efficiently. Therefore, the EGO is very suitable for ship mechanics.

In the field of ship mechanics, ship design optimization belongs to MDO which contains many disciplines, such as structure mechanics, propulsion, resistance, machinery and cost. There is much interactive coupling among these disciplines. Besides, some disciplinary analyses need lots of time to operate, such as CFD for hydrodynamics and FEA for structural analysis. How to find the global optimal point for ship design optimization with MDO in an efficient way is the key point. Since the special framework, Collaborative Optimization (CO), one of the most frequently applied multidisciplinary design optimization methods, happen to be non-convergence to the global optimal point or converging slowly near the global optimal point. EGO will be a good to overcome the weakness, and allows system to converge faster and more robustly. Current work is focused on applying the EGO in CO, which will be a promising

way to improve the efficiency of ship design optimization.

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# 高效优化算法在船舶力学中的应用研究

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摘要:作为一种贝叶斯优化算法,高效全局优化算法(EGO)利用克里格模型来构造近似模型,并采用样本填充准则以寻 找下一个样本点来更新近似模型。文中详细介绍了该优化算法,并将其应用于船舶力学的两个典型优化例子。其中一个 是潜艇的多学科概念设计,考虑了水动力、推进、重量、性能和成本5个学科;另外一个是屈曲状态下加筋板的优化问 题。与传统优化相比,高效全局优化算法不仅收敛到全局最优解,而且更加有效。结果表明高效优化算法非常适用于船 舶力学中的优化问题。

关键词:高效全局优化算法(EGO);船舶力学;克里格模型;样本填充准则(ISC)

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