Mode-coupling analysis of three-dimensional microdisk resonators by the finite-difference time-domain technique

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Quality factor enhancement due to mode coupling is observed in a three-dimensional microdisk resonator. The microdisk, which is vertically sandwiched between air and a substrate, with a radius of 1 \( \mu \text{m} \), a thickness of 0.2 \( \mu \text{m} \), and a refractive index of 3.4, is considered in a finite-difference time-domain (FDTD) numerical simulation. The mode quality factor of the fundamental mode HE\(_{71}\) decreases with an increase of the refractive index of the substrate, \( n_{\text{sub}} \), from 2.0 to 3.17. However, the mode quality factor of the first-order mode HE\(_{72}\) reaches a peak value at \( n_{\text{sub}} = 2.7 \) because of the mode coupling between the fundamental and the first-order modes. The variation of mode field distributions due to the mode coupling is also observed. This mechanism may be used to realize high-quality-factor modes in microdisks with high-refractive-index substrates. © 2006 Optical Society of America

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Semiconductor microdisk lasers\(^1\) have attracted much attention because of advantages such as their ultrasmall volume, high quality factor, and large free spectral range. Many methods have been proposed to investigate their mode properties,\(^2\)–\(^5\) especially for the fundamental mode in the microdisks. In this Letter the mode coupling between the fundamental and the first-order modes in a microdisk vertically sandwiched between air and a substrate is observed by a 3D finite-difference time-domain (FDTD) simulation. For the fundamental mode, we find that the quality factor monotonically decreases with an increase of the refractive index of the substrate, \( n_{\text{sub}} \). But for the first-order mode, the quality factor has a peak value at a certain \( n_{\text{sub}} \) thanks to the mode coupling. The variation in the mode field distributions also verifies the presence of mode coupling. The results show that a mode with a high quality factor can exist in a microdisk with a relatively weak vertical waveguide.

A microdisk resonator with a vertical waveguide formed by the refractive index distribution \( n_{\text{sub}}/n_{\text{disk}}/\text{air} \) is simulated by the 3D FDTD method used in Ref. 4. The microdisk resonator is a pillar structure on a substrate with refractive index \( n_{\text{sub}} \) surrounded by air. To increase the computing efficiency of the FDTD simulation, the 3D problem is transformed to a 2D one in cylindrical coordinates by taking advantage of the azimuthal symmetry of the microdisk with an angular field dependence of \( \exp(i\nu\varphi) \), where \( \nu \) is the azimuthal mode number. Figure 1 shows a cross section of the microdisk with a calculation region bounded by \( \Gamma_a, \Gamma_b, \Gamma_c, \) and \( \Gamma_d \), where \( R \) and \( d \) are the radius and thickness. Mur’s first-order absorbing boundary condition (ABC) is applied on \( E_r \) and \( E_\phi \) field components at the boundaries \( \Gamma_a \) and \( \Gamma_c \), and on \( E_z \) and \( H_z \) field components at the boundary \( \Gamma_d \). The boundaries \( \Gamma_a, \Gamma_c, \) and \( \Gamma_d \) are placed 2.0, 6.0, and 4.0 \( \mu \text{m} \) away from the microdisk’s upper, lower, and lateral boundaries, respectively.

The spatial steps \( \Delta z \) and \( \Delta r \) are set to be 10 and 20 \( \text{nm} \), respectively, and the time step \( \Delta t \) is chosen to satisfy the Courant condition. At the inner boundary \( \Gamma_b \) at \( r = 4\Delta r \), the condition \( \psi_b \propto r^v \) is used for the \( E_z \) and \( H_z \) field components based on the asymptotic behavior of the Bessel function.\(^4\) The refractive index of the microdisk is taken to be \( n_{\text{disk}} = 3.4 \), and the refractive index of the substrate \( n_{\text{sub}} \) is varied from 2.0 to 3.17. In the calculation, an exciting source with a cosine impulse modulated by a Gaussian function \( P(x_0, y_0, t) = \exp[-(t-t_0)^2/t_w^2]\cos(2\pi ft) \) is added to one component of the electromagnetic field at a point \((x_0, y_0)\) inside the microdisk, where \( t_0 \) and \( t_w \) are the times of the pulse center and the pulse half-width, respectively, and \( f \) is the center frequency of the pulse.

Then the time variation of a selected field component at some points inside the microdisk is recorded as the FDTD output. Finally, the Padé approximation with Baker’s algorithm\(^6\) is used to transform the FDTD output from the time domain to the frequency domain and to calculate the mode frequencies and quality factors.

For a microdisk with \( R = 1 \mu \text{m} \) and \( d = 0.2 \mu \text{m} \), the mode wavelength \( \lambda \) and quality factors \( Q \) of the HE\(_{71}\) and HE\(_{72}\) modes are calculated and plotted as functions of \( n_{\text{sub}} \) in Fig. 2. The azimuthal mode number \( \nu \) and the radial mode number \( l \) for HE\(_{\nu l}\) are based on the mode field distributions at \( n_{\text{sub}} = 2.0 \). From the

Fig. 1. Cross section of a microdisk with infinite substrate and the calculation region.
variation of the mode wavelengths with $n_{sub}$ an anticrossing coupling between these two modes is observed at about $n_{sub}=2.45$, similar to that in the discussion of coupled-mode theory. As in quantum mechanical perturbation theory, the mode wavelengths of the HE$_{71}$ and HE$_{72}$ modes split to prevent crossing. The dashed curves are added to explicitly illustrate the mode crossing. Furthermore, the mode quality factor of HE$_{72}$ increases quickly from $1.12 \times 10^{3}$ to $1.05 \times 10^{4}$ as $n_{sub}$ increases from 2.6 to 2.7. In general, an increase in $n_{sub}$ reduces the confinement of the waveguide in the vertical direction. Thus the mode quality factor will decrease with an increase of $n_{sub}$, as will that of the HE$_{71}$ mode. The peak of the quality factor for the HE$_{72}$ mode at $n_{sub}=2.7$ can be attributed to the mode coupling between HE$_{71}$ and HE$_{72}$.

Using a long optical pulse with very narrow bandwidth to excite only one mode, a steady-state distribution of the electromagnetic field can be achieved to exhibit the spatial profile of this mode. Figure 3 depicts the field distribution $H_z$ in the vertical and horizontal directions for the HE$_{71}$ and HE$_{72}$ modes for a microdisk with $R=1.0$ $\mu$m.

Fig. 2. Variation of the mode wavelength $\lambda$ and quality factor $Q$ as functions of the refractive index of the substrate $n_{sub}$ for the HE$_{71}$ and HE$_{72}$ modes for a microdisk with $R=1.0$ $\mu$m.

To verify that the results presented above are general and not a coincidence for a particular waveguide and a certain azimuthal quantum number $v$, we also calculate the mode wavelengths and quality factors for microdisks with $R=1.0$ $\mu$m, $v=6$, and $R=1.2$ $\mu$m, $v=7$, and plot the results in Fig. 4, top and bottom.
respectively. With an increase of $n_{\text{sub}}$, mode wavelength crossings occur at about $n_{\text{sub}}=2.4$ and 2.45 in Fig. 4, top and bottom, respectively. Furthermore, the quality factors of the original first-order mode reach their peak values at about $n_{\text{sub}}=2.6$ and 2.7 due to the mode coupling effect. These results show that the mode coupling between the fundamental and the first-order modes is a rather common phenomenon for 3D microdisk resonators.

It should be noted that the anticrossing coupling between the fundamental and the first-order transverse modes in the microdisk cavity is a novel type of mode coupling, which can be used to enhance the mode quality factor in a microdisk with a high-refractive-index substrate. The most common mode coupling is the coupling between the optical mode and the exciton mode in VCSEL microcavites. In a VCSEL microcavity with an inside grating, mode coupling was also predicted between microcavity and grating modes.

In conclusion, we have investigated the mode coupling between the fundamental and the first-order modes for 3D microdisk resonators. The results show that the enhancement of the quality factor due to the mode coupling can yield a much higher quality factor in a microdisk with a rather weak vertical waveguide when a suitable refractive index of the substrate is chosen.

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