Numerical and Theoretical Analysis of the Crosstalk in Linear Optical Amplifiers

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Abstract—The dynamic characteristics, including the crosstalk and relaxation oscillation, of linear optical amplifiers (LOAs) are investigated by small-signal analysis under an averaging carrier density approximation and compared with the results of numerical simulation. The good agreement between the numerical simulation and the small-signal analysis indicated the averaging carrier density is an appropriate approximation for analyzing LOAs. Theoretical analyzes also show that the dynamic properties of the vertical laser fields dominate the dynamic performance of LOAs. Based on the small-signal analysis, a concise equation for the crosstalk under high bit rate was derived, which can be applied to measure the differential gain of LOAs.

Index Terms—Crosstalk, linear optical amplifier (LOA), relaxation oscillation, semiconductor optical amplifier (SOA), small-signal analysis.

I. INTRODUCTION

THE linear optical amplifier (LOA), which achieves gain clamping by integrating vertical-cavity lasers (VCLs) on the direction perpendicular to the cavity, was introduced recently [1]. It was demonstrated that LOAs would be hopefully implemented into the wavelength-division multiplexing (WDM) networks [2]–[4]. Great advantages, such as low crosstalk, no switch transient [1], low noise figure and wide-band gain clamping [5], have been studied experimentally and theoretically for LOAs.

Mathematical models are required to help the design of LOAs and to predict their operational characteristics. We have developed detailed models and algorithms to simulate the state-steady properties of LOAs [5] and gain-clamped semiconductor optical amplifiers (GCSOAs) [6]. Dynamic simulations for the relaxation oscillation of LOAs were introduced in [7]. However, the dynamic simulation of LOAs is still a time consuming task. It would be advantageous if a simple and convenient method was developed to theoretically predict the properties of LOAs.

For semiconductor lasers, the small-signal analysis is a powerful method to predict the dynamic performance of the lasers. However, the significant longitudinal carrier spatial inhomogeneity as well as gain saturation prevents the small-signal analysis being used for the analysis of traditional semiconductor optical amplifiers (SOAs). An approximate approach, which phenomenologically took into account the inhomogeneous gain, was developed for the small-signal analysis of SOAs [8]. The longitudinal carrier density of LOAs keeps nearly constant before gain saturation [5], so we can expect that the averaging carrier densities will be a reasonable approximation for dynamic analysis of LOAs.

In this paper, we propose a small-signal method using averaging carrier density approximation based on an averaging photon density model [9] to quantitatively investigate the dynamic characteristics of LOAs, and numerically simulate the dynamic performance of LOAs including the crosstalk and relaxation oscillations. The good agreement between the numerical results and small-signal analysis demonstrates that averaging carrier densities is an appropriate approximation for the small-signal analysis of LOAs. The theoretical results also show that the dynamic characteristics of LOAs are greatly dominated by the properties of vertical cavity lasers.

II. DYNAMIC MODEL

In this section, we develop a detailed dynamic model for LOAs based on the steady-state model of LOAs [5] and the dynamic model of SOAs [10]. As a matter of convenience, the rate equations are written down in the form of photon densities as follows:

\[
\frac{\partial n(z,t)}{\partial t} = \frac{I}{qV} - R(z,t) - v_g g(z,t,\nu_{th}) S_{total}(z,t) - v_g \sum_{\nu} g(z,t,\nu) [S^+(z,t,\nu) + S^-(z,t,\nu)]
\]

\[
\frac{\partial S_{total}(z,t)}{\partial t} = v_g \Gamma [g(z,t,\nu_{th}) - g_{th}] S_{total}(z,t) + \Gamma R_{sqL}(z,t,\nu) + \Gamma R_{spL}(z,t,\nu)
\]

\[
\frac{\partial S^\pm(z,t,\nu)}{\partial t} = v_g [\Gamma g(z,t,\nu) - \alpha] S^\pm(z,t,\nu) + \Gamma R_{sq}(z,t,\nu)
\]

where \( \nu \) is the photon frequency, \( \nu_{th} \) is the frequency of the laser field, \( S^\pm \) are the forward and backward propagating photon densities of the signal and spontaneous emission, \( S_{total} \) is the photon density of the lasing light in VCLs, \( R_{sq} \) and \( R_{spL} \) represent the spontaneous emission densities coupled into the amplified spontaneous emission (ASE) and the vertical laser field, \( g \) is the material gain coefficient, \( V \) is the volume of the active region, \( \Gamma \) is the optical confinement factor, \( \Gamma_L \) is the optical confinement factor of the vertical cavity laser (VCL), \( v_g \) is the group velocity,

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\( g_\text{th} \) is the threshold gain of the VCL. The bandgap shrinkage coefficient is used in the numerical solutions. The recombination rates can be written as [5]

\[
R_{\text{sp}}(\nu, n) = 2 \frac{\Gamma}{4 \pi \nu} E_{\text{st}}(\nu, n) \Delta \nu
\]

where \( \nu \) and \( d \) are the width and thickness of the active region, respectively, and \( \Delta \nu \) is the frequency interval of the ASE spectrum. \( R_{\text{sp}, \text{L}}(\nu, n) \) of the vertical cavity laser can take the same form as

\[
R_{\text{sp}, \text{L}}(\nu, n) = 2 \frac{\Gamma}{4 \pi \nu} E_{\text{st}}(\nu, n) \Delta \nu_{\text{m}}
\]

where \( \Delta \nu_{\text{m}} \) is the longitudinal mode interval of the VCL.

We simply take the material gain spectrum of bulk materials for simulation [11], [12] as follows:

\[
g_0(\nu, n) = \frac{c^2}{4\sqrt{2\pi^3/3\eta_1^2 \tau_\text{e} \nu^2}} \left[ \frac{2m_e m_{\text{hh}}}{\hbar (m_e + m_{\text{hh}})} \right]^{3/2} \times \sqrt{\nu - \frac{E_g(n)}{\hbar}} \left[ f_c(\nu) - f_v(\nu) \right]
\]

where \( f_c(\nu) \) and \( f_v(\nu) \) are the Fermi–Dirac distributions in the conduction band and valence band, \( \tau_1 \) is the radiative carrier recombination lifetime, \( \tau_\text{e} \) is defined by \((B_{\text{sp}, \text{off}} n)^{-1}\), and \( E_g(n) \) is the bandgap energy with the following bandgap shrinkage [13]:

\[
E_g(n) = E_{g0} - K y n^{1/3}
\]

where \( K_y \) is the bandgap shrinkage coefficient, and \( E_{g0} \) is the bandgap energy without the injected carriers.

The material gain can also be expressed as the stimulated emission minus the stimulated absorption as follows:

\[
g_0(\nu, n) = E_{\text{st}}(\nu, n) - E_{\alpha}(\nu, n)
\]

where \( E_{\text{st}} \) and \( E_{\alpha} \) are the rates per unit length of stimulated emission and absorption. \( E_{\text{st}} \) can be written as

\[
E_{\text{st}}(\nu, n) = g_0(\nu, n) \frac{f_c(\nu)(1 - f_v(\nu))}{f_v(\nu) - f_c(\nu)}.
\]

The spontaneous term \( R_{\text{sp}}(\nu, n) \) can be written as [5]

\[
R_{\text{sp}}(\nu, n) = 2 \frac{\Gamma}{4 \pi \nu} E_{\text{st}}(\nu, n) \Delta \nu
\]

where \( \nu \) and \( d \) are the width and thickness of the active region, respectively, and \( \Delta \nu \) is the frequency interval of the ASE spectrum. \( R_{\text{sp}, \text{L}}(\nu, n) \) of the vertical cavity laser can take the same form as

\[
R_{\text{sp}, \text{L}}(\nu, n) = 2 \frac{\Gamma}{4 \pi \nu} E_{\text{st}}(\nu, n) \Delta \nu_{\text{m}}
\]

where \( \Delta \nu_{\text{m}} \) is the longitudinal mode interval of the VCL.

Taking into account the nonlinear gain due to the carrier spectral-hole-burning, we simply rewrite the material gain as [14]

\[
g = g_0(1 - \kappa S_t)
\]

where \( \kappa \) is the gain compression coefficient and \( S_t \) is the total photon density in the cavity of each VCL.

### III. Small-Signal Analysis

In the steady-state simulation, the carrier density along the device’s cavity keeps a nearly constant value before gain saturation [5]. Thus, in dynamic small-signal analysis, we assume that carrier density maintains the same value in the cavity direction approximately. The approximation of averaging photon density...
is the ASE part in the carrier rate equation, we simply choose the relaxation resonance frequency and the damping factor. If the signal photon density be much smaller than lasing light photon density along the longitudinal direction, the relaxation resonance frequency and the damping factor can be approximately written as

\[ 2\gamma_R \approx \frac{1}{\tau} + v_g (a_{\text{sig}} S_{\text{sig}}^0 + a_{\text{las}} S_{\text{las}}^0) - v_g \Gamma L (g_{\text{las}}^0 - g_{\text{th}}) \quad (21) \]

\[ \omega_R^2 \approx -v_g^2 \Gamma L a_{\text{las}} (g_{\text{las}}^0 - g_{\text{sig}}^0 - g_{\text{th}}) S_{\text{las}}^0 - v_g \Gamma L (g_{\text{las}}^0 - g_{\text{th}}) (v_g a_{\text{sig}} S_{\text{sig}}^0 + \frac{1}{\tau}) \quad (22) \]

Equation (20) is a second-order differential equation in the form of over damped vibration, where \( \omega_R \) is the relaxation resonance frequency and \( \gamma_R \) is the damping factor. If the signal photon density be much smaller than lasing light photon density along the longitudinal direction, the relaxation resonance frequency and the damping factor can be approximately written as

\[ 2\gamma_R \approx \frac{1}{\tau} + v_g a_{\text{las}} S_{\text{las}}^0 \]

\[ \omega_R^2 \approx v_g^2 \Gamma L a (g_{\text{th}} + S_{\text{sig}}^0 - g_{\text{las}}^0) S_{\text{las}}^0 \quad (23) \]

which are equal to the relaxation resonance frequency and the damping factor of vertical cavity lasers [15], so the properties of the vertical laser fields will dominate the LOAs’ dynamic characteristics.

Equations (17), (18), and (20) are main results for the LOAs under the small-signal approximation. Solving these equations under different initial conditions, we can conveniently predict the dynamic performance of LOAs.

Taking into account the nonlinear gain in the small-signal analysis, the correction of nonlinear gain will be determined by practical value of \( S_t \). We simply choose the \( S_t \) as the averaging value of whole LOA cavity. The nonlinear gain correction \((1 - \kappa S_t)\) is about 0.9999 as \( S_t \approx 1 \times 10^{23}\text{ m}^{-3} \).

IV. RELAXATION OSCILLATIONS

First, we investigate the relaxation oscillations in a system with two signal channels. The probe signal has a constant input power and the pump signal is a square-wave, which is switched on at \( t = 0\text{ ns} \) and off at \( t = 2\text{ ns} \). We try to describe the modulation process of the relaxation oscillation by the above small-signal analysis. At the rising edge of the square-wave pump signal, the initial conditions are

\[ \delta n(0) = 0, \quad \delta S_{\text{las}}(0) = 0 \]

\[ \delta S_{\text{sig}}(t) = \begin{cases} 0, & t < 0 \\ S_{\text{sig}}^0, & t \geq 0 \end{cases} \quad (24) \]

where we assume that \( \delta S_{\text{sig}} \) keeps the same value after the rising edge of the input square-wave signal.

Solving (20) under these initial conditions, we can get the following results:

\[ \delta S_{\text{las}} = -v_g^2 \Gamma L a_{\text{las}} S_{\text{las}}^0 S_{\text{sig}}^0 \delta S_{\text{sig}} \]

\[ \omega_R^2 \]

\[ \frac{d}{dt} \delta S_{\text{las}} + 2\gamma_R \frac{d}{dt} \delta S_{\text{las}} + \omega_R^2 \delta S_{\text{las}} = -v_g^2 \Gamma L a_{\text{las}} S_{\text{las}}^0 S_{\text{sig}}^0 \delta S_{\text{sig}} \]

\[ \left\{ 1 - \exp(-\gamma_R t) \left[ \cos(\omega_R t) + \frac{2R}{\omega_R} \sin(\omega_R t) \right] \right\} \quad (25) \]
where $\omega_R$ is the actual peak frequency of the resonance, which is slightly less than the $\omega_R$

$$\omega_R = \sqrt{\omega_R^2 - \gamma_R^2} \approx \omega_R. \tag{26}$$

Input $\delta S_{\text{las}}$ into the (18), we can get

$$\delta n = - \frac{Vg\delta S_{\text{las}}}{\omega_R} \exp(-\gamma_R t) \sin(\omega_R t)$$

$$+ \frac{Vg0}{\omega_R} \left( \frac{Vg0\delta S_{\text{sig}}}{\omega_R} \right),$$

$$\cdot \left\{ 1 - \exp(-\gamma_R t) \left[ \cos(\omega_R t) + \frac{\gamma_R}{\omega_R} \sin(\omega_R t) \right] \right\}$$

$$\approx - \frac{Vg0\delta S_{\text{sig}}}{\omega_R} \exp(-\gamma_R t) \sin(\omega_R t). \tag{27}$$

For the falling edge, the solutions will be the minus form of the (25) and (27). The expression of the probe signal can be derived out by $\delta n$

$$\delta S_{\text{cw}} = S_{\text{cw, in}} \delta G = S_{\text{cw, in}} \Gamma L\alpha \exp[(\Gamma g_{\text{cw}} - \alpha) L] n. \tag{28}$$

Taking the pump signal input of $-20$ dBm and a constant input probe signal of $-40$ dBm, we investigate the effect of the pump signal acting on the carrier density, the output probe signal and the lasing light by the small-signal analysis, and plot the results in Fig. 2 as dashed lines. In our model, we choose the lasing wavelength at 1530 nm and the signal wavelength at 1540 nm. The device gain in this situation is about 23 dB. The modulation of the pump signal results in a modulation of device gain. However, as we can see in Fig. 2, because the carrier density changes in a very small range about $0.006 \times 10^{24} \text{ m}^{-3}$, the device gain will change very little. As a result, the probe signal output oscillates in a small range of 0.3 dBm, and the mean value of the probe signal keeps the same in the oscillations of the modulation. For the lasing light, because the lasing light will be greatly affected by the changes of the carrier density near threshold [5], the lasing light drops about $0.04 \times 10^{20} \text{ m}^{-3}$ while the carrier density oscillates in a very small range of $0.006 \times 10^{24} \text{ m}^{-3}$ and the change of the average carrier density cannot be read out in the figure.

Taking $M = 30$, we numerically solve rate (1)–(3) with the pumping square wave turns on and off at $t = 0$ and 2 ns, respectively. In Fig. 2, we also plot the results of numerical simulation as solid lines, where the carrier density and lasing photon density of VCLs are averaging values over $M = 30$ regions of the LOA. The results of small signal analysis are agreement very well with the numerical simulations when we choose the equivalent spontaneous emission factor $\beta = 0.003$ used in (14). For the lasing light, there is a small difference between the numerical and theoretical results, because little difference between the carrier densities will affect the lasing light greatly. According to (21) and (22), $\omega_R$ is $3.2 \times 10^{10}$ s$^{-1}$ and $\gamma_R$ is $3.8 \times 10^9$ s$^{-1}$ in the relaxation oscillations. In our model, the small-signal analysis for the relaxation oscillation is available as the input signal is less than about $-16$ dBm. For a large-signal injection with a square-wave of $-10$ dBm from $t = 0$ to 2 ns and a constant input probe signal of $-40$ dBm, we plot the numerically simulated responses and the results of the small-signal model of LOA in Fig. 3 as the solid lines and the dashed lines, respectively. In this case, the numerical simulation shows that the average carrier density drops about $0.03 \times 10^{24} \text{ m}^{-3}$ due to the large-signal injection, which cannot be predicted by the small-signal model. And the large-signal injection results in the drop of the lasing light of VCLs about $2.5 \times 10^{20} \text{ m}^{-3}$. As the large-signal turn off at $t = 2$ ns, the carrier densities in different regions increase and reach peak values at different times due to the different drop of carrier density at each region, and the photon density of lasing light in different VCL reaches the first peak at different time after $t = 2$ ns.

V. CROSS TALK

To obtain the small-signal response of sinusoidal optical injection, we assume that solutions of all variables have the form of harmonic modulation

$$S_{\text{sig}} = S_{\text{sig}}^0 + S_{\text{sig}}^1 \exp(i\omega t)$$

$$S_{\text{las}} = S_{\text{las}}^0 + S_{\text{las}}^1 \exp(i\omega t)$$

$$n = n^0 + n^1 \exp(i\omega t). \tag{29}$$
The crosstalk in this situation is defined as the normalized change of devices gain divided by the normalized change of the sinusoidal signal injection

\[ \text{Crosstalk} = \left( \frac{\Delta G}{\Delta n} \right) \cdot \left( \frac{S_{\text{Signal}}}{S_{\text{Signal}}} \right). \]  

(30)

Input (29) into (20), we can get the solution

\[ S_{\text{bs}}^{1} = - \frac{\nu g \Gamma L}{\omega R} \cdot n^{1} - \frac{\nu g \Gamma L}{\omega R} \cdot S_{\text{bs}}^{1} \cdot S_{\text{Sig}}^{1}. \]  

(31)

\[ n^{1} = - \frac{\nu g \Gamma L}{\omega R} \cdot \frac{\Delta G}{\Delta n} \cdot \frac{S_{\text{bs}}^{0}}{S_{\text{Signal}}} \cdot \frac{S_{\text{Signal}}^{1}}{S_{\text{Signal}}} \]  

(32)

Input (32) into (30), we can derive out

\[ \text{Crosstalk} = \left( \frac{1}{G} \frac{\partial G}{\partial n} \cdot \frac{S_{\text{bs}}^{0}}{S_{\text{Sig}}^{1}} \right) \cdot \delta n \]

\[ = - \nu g \Gamma L \cdot \frac{S_{\text{bs}}^{0}}{S_{\text{Signal}}} \cdot \frac{S_{\text{Signal}}^{1}}{S_{\text{Signal}}} \nu g \Gamma L \left( g_{\text{bs}}^{0} - g_{\text{bs}}^{1} \right) \]  

(33)

Equation (33) illuminates that the crosstalk of LOAs directly proportionate to the pump signal power and the device gain. When \( \omega \) is equal to \( \omega R \), the crosstalk will reach a peak value.

Fig. 4 compares the results of small-signal analysis with the numerical simulation of the crosstalk. These results also agree very well with the experiment results of intrinsic intermodulation distortion [16]. The results show that the averaging carrier density is an appropriate approximation for the small-signal analysis of LOAs. In Fig. 4 the crosstalk reaches a peak value when the external modulation frequency is equal to the intrinsic resonance frequency \( \omega R \). If we want to reduce the crosstalk at high bit-rates, we should reduce \( \omega R \). \( \omega R \) is 5.0 and 7.8 GHz at injection current of 150 and 250 mA, respectively.

The crosstalk at high bit-rates (> 10 GHz) can be approximately derived from (33)

\[ \text{Crosstalk} \approx \frac{1}{\omega} \nu g \Gamma L \cdot \frac{S_{\text{bs}}^{0}}{S_{\text{Signal}}} \cdot \frac{S_{\text{Signal}}^{1}}{S_{\text{Signal}}} \]  

(34)

For device measurements, assuming there is no much difference for the material gain of the signal and lasing light, we can rewrite (34) as

\[ \text{Crosstalk} = - \frac{1}{\omega} \nu g \Gamma L \cdot \frac{S_{\text{bs}}^{0}}{S_{\text{Signal}}} \cdot \frac{S_{\text{Signal}}^{1}}{S_{\text{Signal}}} \]  

(35)

where \( S_{\text{Signal}} \) presents the output signal density. So the differential gain \( \alpha \) can be measured from the crosstalk in LOAs.

Fig. 5 is the comparison of the small-signal analysis from (34) and the numerical results of the crosstalk. The analytical results agree very well with the results obtained by the numerical simulation in high bit-rates and the error is less than 1 dB. Based on (34), we can clearly see that the crosstalk of LOAs is linear with the input power of the pump signal, which is different with the traditional SOAs [10].

VI. CONCLUSION

We have developed detailed theoretical methods to investigate the dynamic properties of LOAs. It is demonstrated that averaging carrier densities is an appropriate approximation for the small-signal analysis of LOAs. The large lasing power of vertical lasers dominated the dynamic properties of LOAs, which is the main reason for the linear crosstalk. We also give some
useful equations for theoretical and experimental analyzing the characteristics of LOAs.

REFERENCES


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