An optimizing design method for a compact iron shielded superconducting magnet of MRI *

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Abstract
A method is developed for designing a special iron shielded superconducting magnet for MRI in this paper. The shield is designed as an integral part of the cryostat and high permeability and high saturated magnetization iron material is adopted. This scheme will result in a compact iron shielded magnet. In the presented design, the finite element (FE) method is adopted to calculate the magnetic field which is produced by superconducting coils and nonlinear iron material. The FE method is incorporated into the simulated annealing method which is employed for corresponding optimization. Therefore, geometrical configurations of both coils and iron shield can be optimized together here. This method can deal with discrete design variables which are defined to describe cable arrangements of coil cross sections. Detail algorithm of presented design is described and an example for designing 1.5T clinical iron shielded magnet for MRI is shown in this paper.

Key words: MRI magnet, iron shield, superconducting coil, optimizing algorithm, simulated annealing, finite element method

1. Introduction
In clinical MRI system with superconducting magnet, shield is commonly used to avoid environment exposing to the fringe field which is produced by the magnet. The shield is also capable of preventing magnetic substances in the vicinity of the magnet from distorting the homogeneity. Currently, there are two manners of shield which are already used in practical MRI: iron shielding and active shielding.

The iron shielding is accomplished by placing iron around the superconducting

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coils. The iron which forms an external flux return path for the magnetic field produced by the coils will reduce the fringe field of the magnet. Active shielding approach employs superconducting active shielding coils to screen the magnetic field outside the magnet. Active shielding has better shield effect than iron shielding, especially, iron is hard to achieve good shield effect in high field magnet (higher than 1.5T), because it is easily to get magnetically saturated. However for 1.5T (or lower 1.0T and 0.5T) magnet, compare to active shielding, iron shielding saves superconducting cables for shielding and the iron used for shield is much cheaper (1). Additionally, the structure of framework for superconducting coils is much simpler and easier to fabricate in iron shielding magnet than in active shielding magnet. So, adopting iron shield will result in low material and fabrication cost.

In previous designs [2-4], the sizes of iron shielding magnets are usually huge, for example, lengths are over 2.3m and widths are over 2m for 1.5T magnet. Those are caused by the thickness and the placement manner of the iron shield. The thickness of shield for 1.5T magnet is commonly around 0.2m [2]. The shields are installed outside around the magnet [4-6]. However, in clinical MRI, short magnets are desired to reduce the perception of claustrophobia for patients. In this work, a scheme for a compact iron shielded magnet is presented. In this scheme, the iron shield is designed as a part of the wall of the vacuum chamber instead of an independent component outside the magnet in previous designs. In addition to this, high magnetic permeability and high saturated magnetization iron material such as magnetic pure iron or low carbon steel which will greatly reduce the thickness of the shield is adopted for the shield.

In the problem of optimizing the iron shielded superconducting magnet which includes current-carrying coils and ferromagnetic materials, the accurate magnetic field calculation is very important. The magnetic field produced by coils which is not affected by other sources can be easily derived from the Biot-Savart law. However, the ferromagnetic material is more complicated because it is magnetized by every other external source and its magnetization is nonlinear because of its nonlinear susceptibility. As well known, finite element (FE) methods are accurate and widely used to solve this type of problem, they were applied to evaluate the magnetic field generated by ferromagnetic materials of iron shielded magnets in several previous works [7-10], but they were not well incorporated into the optimizing procedure. In addition to FE methods, the equivalent magnetization current method which treated
the magnetization of iron shield as equivalent current was employed to deal with field computations for iron shielded magnet [3,11,12]. A rapid field calculation scheme for the effect of ferromagnetic material was presented by Zhao et al. In this scheme, circular rings are considered as basic units of iron shield and magnetic field produced by the ring could be calculated under the condition of arbitrary external sources [1,13]. These two methods take the advantage of directly calculating the expansion harmonics of the magnetic field generated by the iron shield which would facilitate coils optimization. But lengthy and complicated computations are still unavoidable for the magnetization of nonlinear ferromagnetic materials.

According to the basic requirements of MRI, configuration of coils and the geometry of the shield in the iron shielded magnet for MRI must be optimized, the main goals are to generate homogeneous magnetic field to the extent of a few parts per million (ppm) within the diameter of sensitive volume (DSV) and restrict the fringe field to achieve the effect of shielding. In optimization schemes of most prior works [2,3,7-12], the geometries of iron shields were pre-defined and only the configurations of coils are optimized. It is difficult for those schemes to optimize sizes of the shield which are important for designing a compact iron shielded magnet. The Levenberg–Marquardt method was applied by Zhao et al to optimize geometrical configurations of superconducting and ferromagnetic materials together in hybrid shielding magnets [1]. But all the optimizing parameters had to be treated as continuous design variables in that method. Actually coils are wound by superconducting cables. The axially symmetrical coils have rectangular cross sections which can be defined by arrangements of the cables. To optimize configurations of the coils, it’s better to treat the numbers of turns and layers of cables as design variables. In this study, the simulated annealing (SA) method which can deal with discrete design variables is applied to optimize both configurations of coils and the shield for the iron shielded magnet. Additionally, the SA method which is a global optimization technology can prevent the optimizing results from being trapped into local minimal.

In this work, a scheme is presented for a compact iron shielded superconducting magnet of MRI with main field not higher than 1.5T. An optimal method which combines the FE method and the SA method is described to design this type of
magnet. The geometrical configurations of superconducting coils and iron shield can be optimized simultaneously by this method. In addition, the method is capable of deal with discrete parameter spaces such that numbers of turns and layers of coils can be treated as design variables. The optimal design by the presented method for a clinical 1.5T iron shielded magnet is detailed in this paper.

2. Method

2.1 Scheme of the iron shield

The cryostat of the superconducting magnet for MRI usually comprised three thermal shields. The first shield encloses a helium chamber and the second shield is provided around the outer periphery of the first shield. These two shields are cooled continuously by the cold head and kept at specific temperatures. The third shield enclosing the second one is the outmost shield which is at room temperature. In conventional manner of iron shield, the shield is isolated from the magnet and located outside around the third thermal shield. The structure of the magnet with conventional iron shield is showed in Figure 1(a). In our scheme, a part of the third thermal shield is made of ferromagnetic material which is showed in Figure 1(b), it is designed as the iron shield. Compare with conventional structure, this structure save

Fig 1. Schematic of iron shielded superconducting magnet: (a) A magnet with conventional iron shield. (b) The magnet adapting novel iron shield scheme. 1 - iron shield, 2 - superconducting coil, 3 - the first thermal shield enclosing the helium chamber, 4 - the second thermal shield, 5 - the third thermal shield at room temperature.
the space outside of the magnet for iron shield, so it will result in a more compact iron shielded magnet.

Magnetic pure iron with high magnetic permeability and high saturated magnetization is adopted. Its magnetic shielding effect will be greatly improved with this material comparing to common iron. Thus thickness and weight of the iron shield could be greatly reduced which will lead to a compact magnet. The magnetization curve of the magnetic pure iron used for the shield is shown in Figure 2.

![Magnetization curve](image)

**Fig. 2** The magnetization curve of the magnetic pure iron used

### 2.2. Field calculation

In our optimizing procedure, the FE method is applied to calculate the magnetic field produced by the iron shielded magnet. The 2D finite element analysis could be used because of cylindrically symmetric structure of the magnet. The magnetostatic equation below is used to solve the problem.

\[
\nabla \times \left( \frac{1}{\mu} \nabla \times \vec{A} \right) = \vec{J}.
\]

Where \(A\) is the magnetic vector potential, \(\mu\) is the magnetic permeability and \(J\) is the current density. Based on this equation, the finite element matrix equations can be derived by the Galerkin’s method [14].

The geometry model of the magnet is meshed by second order triangular and quadrilateral finite elements (showed in Figure 3). The sizes of the elements are set...
small enough (about 1mm) to achieve high calculation accuracy.

![Fig 3. The finite element mesh for the model of the iron shielded superconducting magnet.](image)

The sparse matrix equations of this problem are solved by the preconditioned dual conjugate gradient method. In order to take into account the non-linearity magnetic characteristic of iron material, the Newton Raphsan iterative method is used to solve the non-linear equation. Once the vector potential $A$ is obtained, the flux density $B$ can be derived from the equation below:

$$\vec{B} = \nabla \times \vec{A}. \quad (2)$$

The axial magnetic flux density $B_z$ is the component mainly concerned about in MRI. Its expansion in spherical harmonics is

$$B_z = B_n + \sum_{n} A_n \cdot r^n \cdot P_n(\cos \theta) \quad (3)$$

Wherein $r$ and $\theta$ are components of positions of field points in spherical coordinates, $\phi$ is absent in this equation because of the cylindrically symmetric structure of the magnet. After the axial flux density $B_z$ in DSV is obtained, expansion coefficients $A_n$ can be calculated by applying linear regression method. In our calculation of the Eq. (3), spherical harmonics not higher than $12^{th}$ order are considered because items of higher order are neglectable.

2.3. Optimizing
The geometrical configurations of the iron shielded superconducting magnet are set as design variables, which is showed in the schematic (Figure 4) of the geometry model of the magnet. The structure of three coil pairs is adapted in our design. The continuous variables \( y_i (i = 1,2,3,4) \) are the sizes and locations of the iron shield. The continuous variables \( x_i (i = 1,2,\cdots,6) \) are the locations of the coils. The turns \( n_i (i = 1,2,3) \) and layers \( m_i (i = 1,2,3) \) of the coils are set as discrete variables. The length and width of the magnet can be controlled by setting constraints for the variables \( x_i \) and \( y_i \).

![Fig 4. The geometry model of the magnet and design variables setting.](image)

The SA method introduced by Metropolis et al, is usually applied to search for optimal arrangement of elements in large scale. It has been used to optimize gradient coils and superconducting magnets of MRI [15,16]. The SA method simulates the way in which a metal slowly cool to minimal energy state. The SA method is a global optimization technique. It avoids being trapped into a local energy minimal by using the Metropolis accept rule which allows the state energy to increase with a probability linked to Boltzmann statistics during the optimization procedure. Additionally, the SA method takes the advantage of dealing with discrete design variables. An adaptive cooling approach which was presented by Hoffmann et al. is adapted for the annealing cooling scheme in our SA method. Details of the method are described in the reference [17] and not repeated here.
To optimize the iron shielded superconducting magnets, the energy function being
minimized is expressed in terms of the magnet properties:

\[ E = W_1 \cdot (B_0 - B_{\text{obj}}) + \sum_{a=1}^{N} w_a \cdot |A_a| \cdot R_{\text{div}}^a + W_2 \cdot \sum_{m=1}^{M} BS_m + W_3 \cdot V_{\text{coil}} + W_4 \cdot W_{\text{shield}} \] (4)

In this equation, the first item controls the main field of the magnet, in which \( B_0 \) is
the calculated main field, \( B_{\text{obj}} \) is the objective main field (1.5T). The second item
controls the homogeneity in terms of field expansion given in Eq. (3), wherein \( R_{\text{div}} \)
is the diameter of the DSV. The fringe field is controlled by the third item which sums
the field values \( BS_m \) at several points along lines outside the magnet, a 5 gauss field
is desired at those lines. The fourth and fifth items constrain the winding volume of
the superconducting wire \( V_{\text{coil}} \) and the weight of the iron shield \( W_{\text{shield}} \) respectively.
\( W_1, W_2, W_3, W_4 \) and \( w_a \) are weighting factors for the items above.

3. Results

The method described above is implemented in C code and run on a HP
workstation. A 1.5T iron shielded superconducting magnet is designed by applying
the proposed method. To design a compact magnet, the magnetic pure iron is used and
a novel structure described above is adapted for the iron shield.

The total length of the magnet is 1720 mm and the width is 1880 mm, the DSV is a
sphere with 40 mm diameter in the center of the magnet, and the diameter of the room
temperature bore is 900 mm. The optimizing results are shown in Figure 5. Figure 5
(a) gives the \( B_0 \) field distribution in the DSV, the peak to peak inhomogeneity in the
DSV is 2.9ppm. Figure 5 (b) gives the fringe field distribution outside the magnet, the
5 Guass line is 5m in the axial direction and 3.5m in the radial direction. The weight
of the iron shield is 14 ton. Table 1 lists optimized values of the design variables.
Table 1. Optimized value of design variables

<table>
<thead>
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<th>variable</th>
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<td>$N_4$ (layers)</td>
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<td>$Y_3$ (mm)</td>
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The best so far energy value at each cooling step in the optimizing procedure is shown in Figure 6. It clearly illustrates that the system energy decreases as the system cools down and tends to converge to the global optimal value after 120 steps of cooling down.
In this work, a novel scheme of iron shield for the superconducting magnet of MRI is presented. The scheme combined with adapting high magnetic permeability and high saturated magnetization iron material to improve shielding effect will result in more compact iron shielded superconducting magnet. To design this type of magnet, an optimizing design method is described in this paper. In this method SA is applied for optimization and FE is used for field calculation, which takes the advantage of optimizing the configurations of the coils and the shield simultaneously. The design results of the clinical 1.5T superconducting magnet detailed in the paper demonstrates the efficiency of the proposed method.

References


